Heterogeneous Ambiguity and Intermediary Asset Pricing

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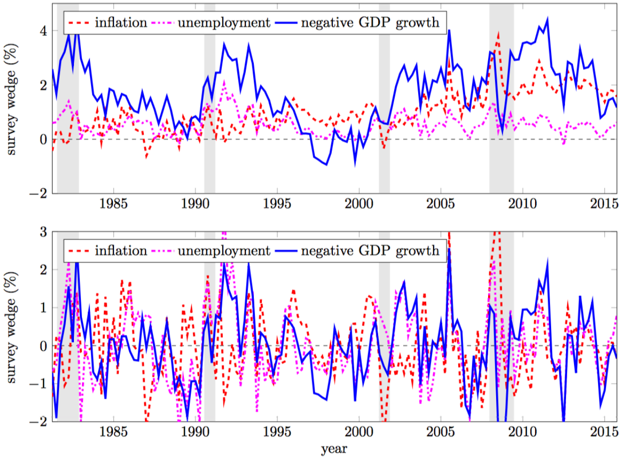
Introduction

- Bhandari, Borovicka and Ho (2016)



Introduction

- Difference in survey expectations between the Michigan Survey and Survey of Professional Forecasters. Top panel original data, bottom panel HP-filtered and standardized. GDP growth forecast for the Michigan Survey is constructed using a projection on the Index of Consumer Expectations, and the GDP growth wedge is plotted with a negative sign. NBER recessions shaded.
- Households' expectations are systematically pessimistically biased – relative to professional forecasters
- Three time series for the belief wedges have a common business cycle component and are statistically significantly correlated.

Motivation

- Intermediary capital can affect asset prices.
- Robustness(RB) or model uncertainty influence investors' portfolio choices and asset prices.
- Household has different RB preference than intermediary specialist.

What Does This Paper Do

- A general equilibrium model of segmented markets with intermediation.
- In the crisis of complex assets.
- Heterogenous robustness preferences of intermediaries and households.
- Financial frictions and economic crisis.
- Mechanism: Robustness affects risk-sharing therefore intermediary portfolio choice and asset prices; also influences the critical value of wealth through participation which incures financial constraint.

Model

The economy.

- Framework: He and Krishnamurthy (2012, RES)
- Intermediation: short-term contract between agents; Equilibrium in competitive intermediation mkt
- Asset pricing: optimal consumption and portfolio decision

Agents and Assets

- Infinite horizon continous time Lucas (1978) tree model.
- Risky asset with dividend follows GBM

$$\frac{dD_t}{D_t} = gdt + \sigma dZ_t \tag{1}$$

- Riskless asset in zero-net supply with interest rate r.
- Risky asset price P_t is determined in general equilibrium (GE).
- Total return on risky asset is

$$dR_t = \frac{D_t dt + dP_t}{P_t} = \mu_{R,t} dt + \sigma_{R,t} dZ_t \tag{2}$$

- Define risky asset risk premium

$$\pi_{R,t} \equiv \mu_{R,t} - r$$

- Households maximizes

$$\mathbb{E}\left[\int_0^\infty e^{-\rho^h t} \ln C_t^h dt\right]$$

  - No participation in risky asset mkt. Only through intermediaries.
- Specialist maximizes

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t} \ln C_t dt\right]$$

  - Only specialists(in charge of intermediary) can invest in risky asset mkt.
- Contracting between two agents due to moral hazard problem.

Intermediation Contract

- One period principle agent problem; two stage game.
- HH wealth W_t^h, contributes T_t^h as equity investment to intermediaries; $W_t^h - T_t^h$ directly to riskless bond.
- Specialist wealth W_t, all to intermediaries.
- Intermediary capital $T_t^I = T_t^h + W_t$ with ε_t^I into risky asset and $1 - \varepsilon_t^I$ into riskless bond.
- A share β_t is specified by contracting of risky asset return goes to specialist.
  - Specialist net exposure: $\varepsilon_t^* \equiv \beta_t \varepsilon_t^I$
  - HH net exposure: $\varepsilon_t^h = (1 - \beta_t)\varepsilon_t^I = \frac{1-\beta_t}{\beta_t}\varepsilon_t^*$

Intermediation Contract

- Sign a contract at t, perish at $t + dt$.
- Unobserved due diligence action $s_t = 0, 1$. Shirking ($s_t = 1$) reduces return by X_t but brings private benefit B_t.
- Unobserved portfolio choice
- Intermediary total return: $\varepsilon_t^I (dR_t - rdt) + T_t^I rdt - X_t s_t dt$; private benefit $s_t B_t dt$.
- Dynamic budget constraint

$$\begin{aligned} dW_t = & \quad rW_t dt - C_t dt + \beta_t \varepsilon_t^I (dR_t - rdt) + K_t dt \\ dW_t^h = & \quad rW_t^h dt - C_t^h dt + (1 - \beta_t) \varepsilon_t^I (dR_t - rdt) - K_t dt \end{aligned}$$

- Effective transfer $K_t \equiv \left(\beta_t T_t^I - W_t \right) r + \hat{K}_t dt$.
- Define per-unit exposure price $k_t \equiv \frac{K_t}{\varepsilon_t^h}$.

Incentive Contraint and Equity Implementation

- Contract (β_t, K_t)
- IC constraint: No shirking: $s_t = 0$

$$\beta_t \geq \frac{B_t}{X_t} \equiv \frac{1}{1+m} < 1 \tag{3}$$

- m reflects the financial constraint due to agency frictions.
- Risk-sharing Constraint

$$\varepsilon_t^h \leq m\varepsilon_t^* \tag{4}$$

HH Consumption/Portfolio Rules

- HH objective:

$$\max_{\{C_t, \varepsilon_t^h\}} \mathbb{E}\left[\int_0^\infty e^{-\rho^h t} \ln C_t^h dt\right] \quad (5)$$

$$s.t.\, dW_t^h = \varepsilon_t^h (dR_t - rdt) - k_t \varepsilon_t^h dt + W_t^h rdt - C_t^h dt \quad (6)$$

- Optimal consumption and portfolio rule

$$C_t^{h*} = \rho^h W_t^h \quad (7)$$

$$\varepsilon_t^{h*} = \frac{\pi_{R,t} - k_t}{\sigma_{R,t}^2} W_t^h \quad (8)$$

Specialist Consumption/Portfolio Rules

- Specialist objective:

$$\max_{\{C_t,\varepsilon_t,\beta_t\}} \mathbb{E}\left[\int_0^\infty e^{-\rho t} \ln C_t dt\right] \tag{9}$$

$$s.t.\, dW_t = \varepsilon_t(dR_t - rdt) + \max_{\beta_t \in [\frac{1}{1+m},1]} \left(\frac{1-\beta_t}{\beta_t}\right) k_t \varepsilon_t^* + W_t r dt - C_t dt \tag{10}$$

- Exposure supply schedule: $\beta_t^* = \frac{1}{1+m}$ if $k_t > 0$; $\beta_t^* \in \left[\frac{1}{1+m}, 1\right]$ if $k_t = 0$
- Optimal consumption and portfolio rule

$$C_t^* = \rho W_t \tag{11}$$

$$\varepsilon_t^* = \frac{\pi_{R,t}}{\sigma_{R,t}^2} W_t \tag{12}$$

- Define per-unit of specialist fee: $q_t \equiv K_t / W_t = \left(\frac{1-\beta_t^*}{\beta_t^*}\right) k_t \frac{\pi_{R,t}}{\sigma_{R,t}}$.

- Define the scaled specialist wealth $x_t \equiv W_t / D_t$ as the aggregate state
- Y is a function of x_t

$$dY(x_t) = \mu_{Y,t} dt + \sigma_{Y,t} dZ_t \tag{13}$$

- where

$$\mu_{Y,t} \equiv Y'(x_t)\mu_{x,t} + \frac{1}{2} Y''(x_t)\sigma_{x,t}^2 \tag{14}$$

$$\sigma_{Y,t} \equiv Y'(x_t)\sigma_{x,t} \tag{15}$$

HH Robust Consumption/Portfolio Rules

- Incorporating Model Uncertainty due to Robustness
- HH problem:
- Take equation (6) as approximating model. The corresponding distorting model can thus be obtained by adding an endogenous distortion $v_t^h = \begin{bmatrix} v_{1,t}^h \\ v_{2,t}^h \end{bmatrix}$:

$$dW_t^h = \left(\varepsilon_t^h(\pi_{R,t} - k_t) + rW_t^h - C_t^h\right) dt + \sigma_{R,t}\varepsilon_t^h \left(\sigma_{R,t}\varepsilon_t^h v_{1,t}^h dt + dZ_t\right) \tag{16}$$

$$dY^h(x_t^h) = \mu_{Y,t}^h dt + \sigma_{Y,t}^h(\sigma_{Y,t}^h v_{2,t}^h dt + dZ_t)$$

- Choose drift adjustment v_t^h to minimize the sum of the expected continuation payoff, but adjusted to reflect the additional drift component in (16), and of entropy penalty:

$$\inf_{v_t^h} \left[DV(W_t^h, Y_t^h) + (v_t^h)^\top \cdot \Sigma^h \cdot \partial V(W_t^h, Y_t^h) + \frac{1}{\theta_t^h}\mathscr{L}\right] \tag{17}$$

$$\inf_{v_t^h}\left[DV(W_t^h,Y_t^h)+(v_t^h)^\top\cdot\Sigma^h\cdot\partial V(W_t^h,Y_t^h)+\frac{1}{\theta_t^h}\mathscr{L}\right]$$

$$DV(W_t^h,Y_t^h)=V_w\left[\varepsilon_t^h(\pi_{R,t}-k_t)+rW_t^h-C_t^h\right]+\frac{1}{2}V_{ww}(\varepsilon_t^h)^2\sigma_{R,t}^2+\mu_{Y,t}^h$$

- Relative entropy (or expected log likelihood ratio between the distorted model and the approximating model which measures the distance between the two models)

$$\mathscr{L}=\frac{(v_t^h)^\top\cdot\Sigma^h\cdot v_t^h}{2}$$

- $\frac{1}{\theta_t^h}$ is the weight on the entropy penalty term and

$$\Sigma^h=\begin{bmatrix}(\varepsilon_t^h\sigma_{R,t})^2 & \varepsilon_t^h\sigma_{R,t}\sigma_{Y,t}^h\\ * & (\sigma_{Y,t}^h)^2\end{bmatrix}$$

- HH solves the following HJB equation s.t.(16):

$$\sup_{\{C_t^h,\varepsilon_t^h\}} \inf_{v_t^h} \left[\ln C_t^h - \rho^h V + DV + (v_t^h)^\top \cdot \Sigma^h \cdot \partial V + \frac{1}{\theta_t^h} \mathscr{L} \right] \tag{18}$$

- Solving first the infirmization part yields

$$v_t^{h*} = \begin{bmatrix} -\theta_t^h V_w \\ -\theta_t^h \end{bmatrix} \tag{19}$$

- Substituting for v_t^{h*} in the HJB equation gives

$$0 = \sup_{\{C_t^h,\varepsilon_t^h\}} \left[\ln C_t^h - \rho^h V + DV - \frac{\theta_t^h}{2} (\sigma_{R,t} \varepsilon_t^h V_w + \sigma_{Y,t}^h)^2 \right] \tag{20}$$

- Optimal HH consumption and portfolio rule under RB are

$$C_t^h = \frac{1}{V_w} \tag{21}$$

$$\varepsilon_t^h = -\frac{V_w(\pi_{R,t} - k_t - \theta_t^h \sigma_{Y,t}^h \sigma_{R,t})}{\sigma_{R,t}^2 V_{ww} - \theta_t^h \sigma_{R,t}^2 V_{ww}^2} \tag{22}$$

- Guess value function takes the form $V(W_t^h, Y_t^h) = A^h \ln W_t^h + Y^h(x_t^h)$
- Finally, $A^h = 1/\rho^h$ and $Y^h(x_t^h)$ satisfies the following ODE (for simplicity, I dropped the time script):

$$\mu_Y^h = \rho^h Y^h - \ln \rho^h - \frac{r}{\rho^h} + 1 + \frac{\theta^h}{2}(\sigma_Y^h)^2 + \frac{(\pi_R - k - \theta^h \sigma_Y^h \sigma_R)^2}{2\sigma_R^2(\rho^h + \theta^h)} \tag{23}$$

Specialist Robust Consumption/Portfolio Rules

- Specialist problem:
- Take equation (10) as approximating model. The corresponding distorting model can thus be obtained by adding an endogenous distortion $v_t = \begin{bmatrix} v_{1,t} \\ v_{2,t} \end{bmatrix}$:

$$dW_t = (\varepsilon_t \pi_{R,t} + (q_t + r) W_t - C_t)\, dt + \sigma_{R,t} \varepsilon_t (\sigma_{R,t} \varepsilon_t v_t dt + dZ_t) \quad (24)$$

$$dY(x_t) = \mu_{Y,t} dt + \sigma_{Y,t} (\sigma_{Y,t} v_{2,t} dt + dZ_t)$$

- Choose drift adjustment v_t to

$$\inf_{v_t} \left[DJ(W_t, Y_t) + v_t^\top \cdot \Sigma \cdot \partial J(W_t, Y_t) + \frac{1}{\theta_t} \mathscr{H} \right] \quad (25)$$

$$\inf_{v_t}\left[DJ(W_t,Y_t)+v_t^{\top}\cdot\Sigma\cdot\partial J(W_t,Y_t)+\frac{1}{\theta_t}\mathscr{H}\right]$$

$$DJ(W_t,Y_t)=J_w\left[\varepsilon_t\pi_{R,t}+(q_t+r)W_t-C_t\right]+\frac{1}{2}J_{ww}\varepsilon_t^2\sigma_{R,t}^2+\mu_{Y,t}$$

- Relative entropy

$$\mathscr{H}=\frac{v_t^{\top}\cdot\Sigma\cdot v_t}{2}$$

- $\frac{1}{\theta_t}$ is the weight on the entropy penalty term and

$$\Sigma=\left[\begin{array}{cc}\varepsilon_t^2\sigma_{R,t}^2 & \varepsilon_t\sigma_{R,t}\sigma_{Y,t}\\ * & \sigma_{Y,t}^2\end{array}\right]$$

- Specialist solves the following HJB equation s.t.(24):

$$\sup_{\{C_t,\varepsilon_t\}} \inf_{\nu_t} \left[\ln C_t - \rho J + DJ + +\nu_t^{\top} \cdot \Sigma \cdot \partial J + \frac{1}{\theta_t} \mathscr{H} \right] \tag{26}$$

- Solving first the infirmization part yields

$$\nu_t^* = \begin{bmatrix} -\theta_t J_w \\ -\theta_t \end{bmatrix} \tag{27}$$

- Substituting for ν_t^{h*} in the HJB equation gives

$$0 = \sup_{\{C_t,\varepsilon_t\}} \left[\ln C_t - \rho J + DJ - \frac{\theta_t}{2} (\sigma_{R,t} \varepsilon_t J_w + \sigma_{Y,t})^2 \right] \tag{28}$$

- Optimal specialist consumption and portfolio rule under RB are

$$C_t = \frac{1}{J_w} \tag{29}$$

$$\varepsilon_t = -\frac{J_w(\pi_{R,t} - \theta_t \sigma_{Y,t} \sigma_{R,t})}{\sigma_{R,t}^2 J_{ww} - \theta_t \sigma_{R,t}^2 J_{ww}^2} \tag{30}$$

- Guess value function takes the form $J(W_t, Y_t) = A \ln W_t + Y(x_t)$
- Finally, $A = 1/\rho$ and $Y(x_t)$ satisfies the following ODE:

$$\mu_Y = \rho Y - \ln \rho - \frac{q+r}{\rho} + 1 + \frac{\theta}{2}\sigma_Y^2 + \frac{(\pi_R - k - \theta \sigma_Y \sigma_R)^2}{2\sigma_R^2(\rho + \theta)} \tag{31}$$

Two-agents Robust Optimal Consumption/Portfolio Rules

- Household robust optimal consumption rule is

$$C_t^h = \rho^h W_t^h \tag{32}$$

- and the robust optimal risk exposure is

$$\varepsilon_t^{h*} = \frac{\pi_{R,t} - k_t + \theta^h \sigma \sigma_{R,t} Y_t^{h'} x_t^h}{\sigma_{R,t}^2 (1 + \theta^h/\rho^h + \theta^h Y_t^{h'} x_t^h)} W_t^h \tag{33}$$

- Specialist robust optimal consumption rule is

$$C_t = \rho W_t \tag{34}$$

- and the robust optimal risk exposure is

$$\varepsilon_t^* = \frac{\pi_{R,t} + \theta \sigma \sigma_{R,t} Y_t^{'} x_t}{\sigma_{R,t}^2 (1 + \theta/\rho + \theta Y_t^{'} x_t)} W_t \tag{35}$$

- When $\theta = \theta^h = 0$, drop to the baseline model.

Comparative Analysis

- It can be showed that

$$\frac{\partial \varepsilon_t}{\partial \theta} > 0 \text{ and } \frac{\partial \varepsilon_t^h}{\partial \theta^h} > 0$$

- RB increases the desired risky asset position.

Market Equilibrium

Definition of equilibrium

An equilibrium for the economy is a set of progressively, measurable price processes $\{P_t\}$ and $\{k_t\}$, households' decisions $\{C_t^{h*}, \varepsilon_t^{h*}\}$, and specialists' decisions $\{C_t^*, \varepsilon_t^*, \beta_t^*\}$ such that

1. Given the processes, decisions solve (5) and (9).
2. The intermediation market reaches equilibrium with risk exposure clearing condition,
$$\varepsilon_t^{h*} = \frac{1-\beta_t^*}{\beta_t^*}\varepsilon_t^*. \qquad (36)$$
3. The stock market clears:
$$\varepsilon_t^* + \varepsilon_t^{h*} = P_t. \qquad (37)$$
4. The goods market clears:
$$C_t^* + C_t^{h*} = D_t. \qquad (38)$$

- In equilibrium, from (38),

$$\rho W_t + \rho^h W_t^h = D_t$$

- therefore,

$$x_t^h = \frac{1}{\rho^h} - \frac{\rho}{\rho_h} x_t \tag{39}$$

- Then we can derive all the equilibrium variables as functions of x_t (i.e.x_t is the only state variable).

- From (33) and (35), define the coefficients of ε_t and ε_t^h as $G(x_t;\theta^h)$ and $F(x_t;\theta)$, such that

$$\varepsilon_t^h = G(x_t;\theta^h)W_t^h$$

$$\varepsilon_t = F(x_t;\theta)W_t$$

  - It can be showed that $\partial G(x_t;\theta^h)/\partial\theta^h > 0$ and $\partial F(x_t;\theta)/\partial\theta > 0$.
  - Later, I will find the explicit processes for $G(x_t;\theta^h)$ and $F(x_t;\theta)$.
- Price/Dividends ratio:

$$\frac{P_t}{D_t} = \frac{G(x_t;\theta^h)}{\rho^h} + \left(F(x_t;\theta) - \frac{\rho}{\rho^h}G(x_t;\theta^h)\right)x_t \qquad (40)$$

Exposure Supply and Demand Schedule

- The **specialist exposure supply schedule** is a step function

$$\begin{cases} \frac{1-\beta_t^*}{\beta_t^*}\varepsilon_t^* \in [0, m\varepsilon_t^*], \text{ for any } \beta_t^* \in [\frac{1}{1+m}, 1] & \text{if } k_t = 0, \\ m\varepsilon_t^* \text{ with } \beta_t^* = \frac{1}{1+m} & \text{if } k_t > 0. \end{cases} \tag{41}$$

- with $\varepsilon_t^* = F(x_t; \theta)W_t$
- The **HH exposure demand** is $\varepsilon_t^h = G(x_t; \theta^h)W_t^h$
- Equity capital constraint: $\varepsilon_t^h \leq m\varepsilon_t^*$
- Both exposure supply and demand are influenced by RB.

Unconstrained and Constrained Region Conditions

- In **unconstrained region**, exposure supply>demand, $k_t = 0$, equity capital constraint is slack, such that

$$\varepsilon_t^h|_{k_t=0} < m\varepsilon_t \iff G(x_t;\theta^h)W_t^h < mF(x_t;\theta)W_t$$

  - Intermediary earns higher exposure, so that HH put all the wealth into the intermediation, $T_t^h = W_t^h$.

- In **constrained region**, exposure supply<demand, $k_t > 0$,equity capital constraint is binding, such that

$$\varepsilon_t^h = m\varepsilon_t \iff G(x_t;\theta^h)W_t^h = mF(x_t;\theta)W_t$$

  - In equilibrium, specialist earn a rent $q_t = \frac{k_t m\varepsilon_t^*}{W_t} > 0$ for scarce intermediary service. With $k_t \uparrow$, $\varepsilon_t^{h*} \downarrow$, exposure demand $\downarrow$.

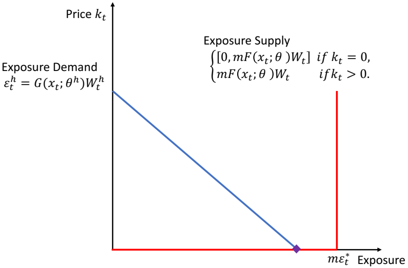
- When the equity capital constraint just starts to bind,

$$x_t^c = \frac{G(x_t;\theta^h)}{\rho^h m F(x_t;\theta) + \rho G(x_t;\theta^h)} \tag{42}$$

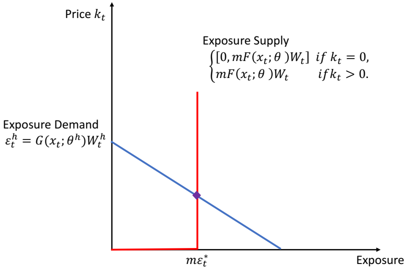
  - Without RB, $x^c = \frac{1}{m\rho^h + \rho}$: $m\uparrow$, $x^c\downarrow$: severity of agency problem.
  - With RB, x_t^c changes due to robust concern through $G(x_t;\theta^h)$ and $F(x_t;\theta)$: $\theta(\theta^h)\uparrow$, $x_t^c\uparrow$.

Unconstrained and Constrained Regions

Unconstrained Region $\varepsilon_t^h|_{k_t=0} < m\varepsilon_t^*$



Constrained Region $\varepsilon_t^h|_{k_t=0} \geq m\varepsilon_t^*$



Specialist Portfolio Choice

- The specialist makes a portfolio choice to invest fraction α_t of the total equity of $T_t^I = W_t + T_t^h$.
- Thus, specialist exposure is $\alpha_t W_t$ (he will choose α_t to set $\alpha_t W_t = \varepsilon_t^*$)
- HH exposure is $\varepsilon_t^h = \alpha_t T_t^h$.
- We can solve that, in the uncontrained region,

$$\alpha_t^U = F(x_t;\theta) + [G(x_t;\theta^h) - F(x_t;\theta)] \frac{1-\rho x_t}{(\rho - \rho^h)x_t + 1} \tag{43}$$

  - m doesn't influence α_t^U.
  - with $\theta = \theta^h = 0$, $G = F = 1$, $\alpha^U = 1$ which coinsides with He and Krishnamurthy (2012).
- In the contrained region,

$$\alpha_t^C = \frac{G(x_t;\theta^h)}{G(x_t;\theta^h) + mF(x_t;\theta)} \left(F(x_t;\theta) - \frac{\rho}{\rho^h} G(x_t;\theta^h)^2 \right) \tag{44}$$

$$+ \frac{G(x_t;\theta^h)^2}{\rho^h [G(x_t;\theta^h) + mF(x_t;\theta)] x_t} \tag{45}$$

Solve for $Y(x_t;)$ and $Y^h(x_t)$ Processes

- Assume $\sigma_x, t = 0$. Utilize one boundary condition: $\alpha_t^U|_{\theta=0} = 1$.
- Guess $Y(x_t)$ takes the form $Y(x_t) = A\exp(Bx_t) + C$, from ODE of $Y(x_t)$ (31) and (23), I solve

$$\begin{cases} A = & \text{any value, suppose 1} \\ B = & \frac{\rho}{\mu_x} \\ C = & \frac{r}{\rho^2} + \frac{\ln\rho - 1}{\rho} - \frac{1}{2\rho^2} \end{cases}$$

- Thus,

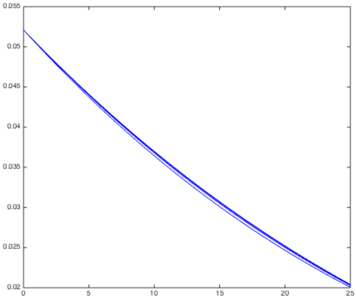
$$Y(x_t) = \exp(\frac{\rho}{\mu_x}x_t) + \frac{r}{\rho^2} + \frac{\ln\rho - 1}{\rho} - \frac{1}{2\rho^2} \quad (46)$$

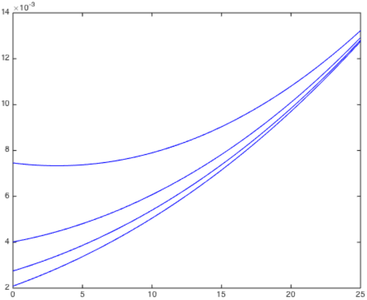
- Similarly,

$$Y^h(x_t) = \exp(\frac{1-\rho}{\mu_x}x_t) + \frac{r}{(\rho^h)^2} + \frac{\ln\rho^h - k_t}{\rho^h} - \frac{1}{2(\rho^h)^2} \quad (47)$$

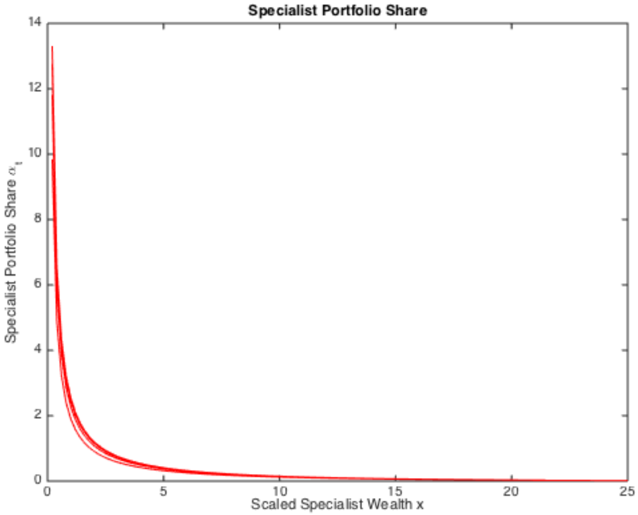
Solve for $G(x_t;\theta^h)$ and $F(x_t;\theta)$ Processes

- Set $G(x_t;\theta^h)$ and $F(x_t;\theta)$ when $\alpha_t = \alpha_t^U$, we can solve them numerically. For simplicity, assume $\rho = \rho^h$,
- Simulation of $G(x_t;\theta^h)$ and $F(x_t;\theta)$:

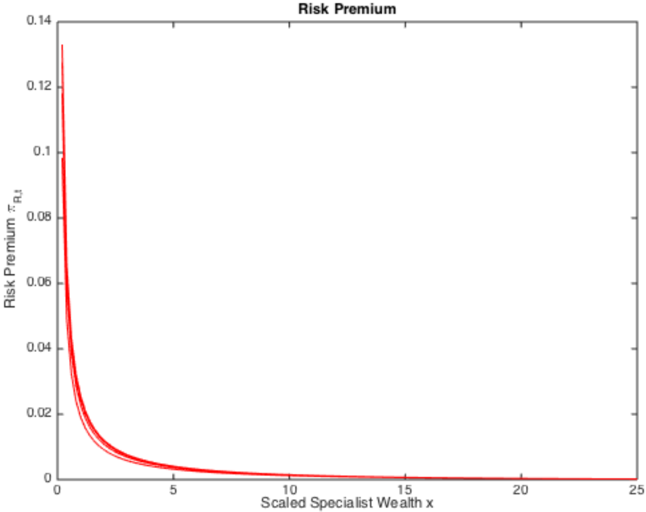




Specialist's Portfolio Share



Risk Premium



Threshold x^c

x^c	θ			
θ^h		0.1	0.2	0.3
	0.1	20.8	21.2	21.4
	0.2	21	21.2	21.4
	0.3	21	21.2	21.4

Conclusion

- A general equilibrium model of segmented markets with intermediation.
- Higher robust preference from HH and specialist both increases optimal portfolio choice and asset prices;
- Both RBs increases the threshold value of wealth through participation which incures equity capital constraint.
- Heterogenous robust preferences of intermediaries and HH play different roles. HH RB influence more than specialist's RB.
- Severely in financial crisis under the existence of equity capital constraint.

Thank You!